Asset Liability Management

Remember for studying the risk we always try to concentrate on the losses rather than on the income/gains. With this view we can formulate a optimization problem such that:

$$Min \sum Un\exp ectedLoss$$

such that:

$$\sum PortfolioWeights = 1$$

$$\sum (ExpectedIn terestInco me - ExpectedLo ss) \geq T\arg et\,\mathrm{Re}\, turnonEqui ty$$

$$\sum TotalAssets \leq \frac{KnownIncome}{ROA}$$

Asset Liability Management

Doing **post optimality analysis** is helpful in identifying the factors and their marginal cost that constrain the institution from reaching goals superior to those indicated by the optimal solution.

In evaluating credit risk, the ultimate measure is the risk of the entire portfolio. We can easily form portfolios such that the overall risk of the portfolio is substantially less than the stand alone risks (of the individual assets). This is due to diversification. Our main aim in portfolio management is not to determine the overall risk alone, but to determine how to achieve the maximal return for that risk that is being taken or assumed.

Asset Liability Management

Three key points emerge from portfolio analysis

1) The amount of diversification achievable in a portfolio depends on the correlations between default risks with the portfolio.
2) The amount of risk contributed by an asset to the portfolio's net diversification depends significantly on how much of the asset is held in the portfolio.
3) Improving portfolio performance consists of including large numbers of assets and varying asset holdings to bring each asset's contribution to the portfolio risk into line with its contribution to the portfolio return.

Asset Liability Management

Altman's Portfolio Approach

$$Max\left(\frac{\bar{r}_P}{\sigma_P} = \frac{\sum_{i=1}^{n} w_i \bar{r}_i}{\sqrt{\sum_{i,j=1}^{n} w_i w_j \sigma_{ij}}}\right)$$

such that

$$\sum_{i=1}^{n} w_i = 1$$

$$\bar{r}_P \geq \bar{r}^*$$

$X_i \leq$ individual investment limit. Remember here $w_i = X_i/\{$Total Amount available for investment$\}$

Asset Liability Management

An alternative formulation of the problem discussed

$$Min\left(\frac{\sigma_P}{\bar{r}_P} = \frac{\sqrt{\sum_{i,j=1}^{n} w_i w_j \sigma_{ij}}}{\sum_{i=1}^{n} w_i \bar{r}_i}\right)$$

such that

$$\sum_{i=1}^{n} w_i = 1 \qquad \sum_{i=1}^{n} w_i \bar{r}_i \,(= \bar{r}_P) \geq \bar{r}^*$$

$$\sum_{i,j=1}^{n} w_i w_j \sigma_{ij} \leq \sigma^{2*}$$

$X_i \leq$ individual investment limit. Remember here $w_i = X_i$/{Total Amount available for investment}

Asset Liability Management

Morgan and Gollinger

$$Min \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} \left(= \sum_{i}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i>j}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}\right)$$

such that

$$\sum_{i=1}^{n} w_i = 1$$

$$\sum_{i=1}^{n} w_i \bar{r}_i \geq \bar{r}^*$$

$0 < w_i < 0.1$

Asset Liability Management

Alternative formulation of Morgan and Gollinger's optimization problem.

$$Min \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} \left(= \sum_{i}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i>j}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}\right)$$

such that

$$\sum_{i=1}^{n} w_i = 1 \qquad \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} \leq \sigma^{2*} \qquad \sum_{i=1}^{n} w_i \bar{r}_i \geq \bar{r}^*$$

$0 < w_i < 0.1$

Asset Liability Management

For any portfolio we can use the concept of unexpected loss where by definition unexpected loss is as given below.

$$UAL_P = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} - \sigma^{2*}$$

Logically we will always try to make it as minimum as possible if not zero. The value of σ^{2*} is dependent on the percentage error we want to maintain, how risk averse we are, the business we are into, the profile of assets which have been taken into consideration for making the portfolio, etc.

Asset Liability Management

Generally in ALM we use two variables to study the credit default situation for any FI and they are:

1) Interest income
2) NPV of {Assets – Liabilities}

The main difference between Interest income and NPV are:

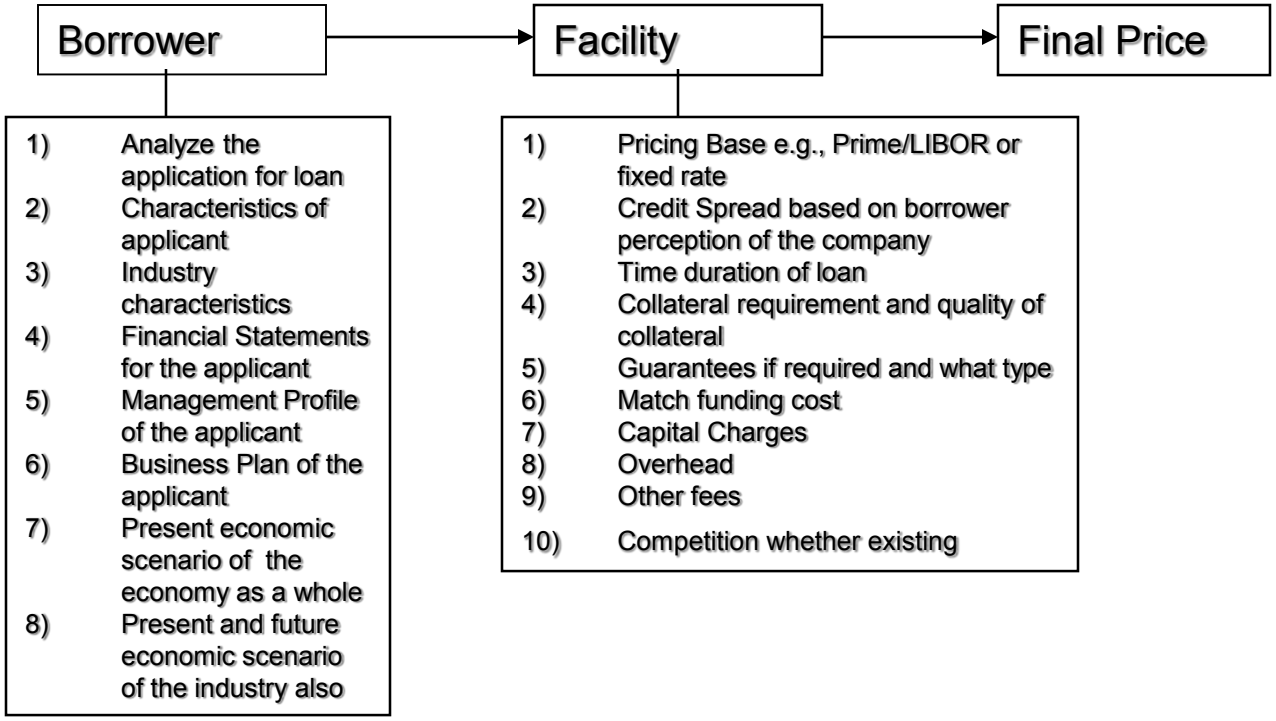
- NPV captures the entire stream of future cash flows (both in flow as well as out flow) generated by the portfolio
- While for interest income we are mainly interested in calculating the inflows rather than the outflows. The reason can be, we think that the profile of the outflows would not be changing substantially, hence we should be concentrating more on the inflows.

Asset Liability Management

Building blocks of ALM

1) Risk drivers: What are the main risks which the bank is facing while modeling its risk exposure. Is it foreign currency risk, interest risk, country risk, etc?
2) Risk exposure: For which risk is the time factor most important, such that the variation in the time has the maximum effect on the risk exposure.
3) Standalone Risk: Valuation of adverse deviations of interest margin and NPV[Assets – Liabilities]. For banks interest rate (hence interes margin) and foreign currency rates have the maximum adverse affect so as to trigger a default.
4) Correlations: Between portfolios and the different types of financial transactions. By a portfolio we would mean any loan or investment the bank has made. For our convenience we can term this as an asset also.
5) Portfolio Risk: What is the distribution of the inflow and the outflow and how far do they match in terms of the time frame. We are interested in finding whether time mismatch is there or not.
6) Capital: In general we are interested in matching the assets and the liabilities so as to maintain a permissible value for the gap.

Asset Liability Management



Asset Liability Management

Transition matrix

Initial Rating	Rating at year end		
	A	B	C
A	0.90	0.08	0.02
B	0.10	0.85	0.05
C	0.01	0.06	0.93

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Transition matrix

Given a_{ij} we know that P[j|i] = a_{ij}, i.e., given a company is at rating i, the probability it will move to a new rating j, at the end of the stipulated time frame, is given by a_{ij}.

Remember the following:

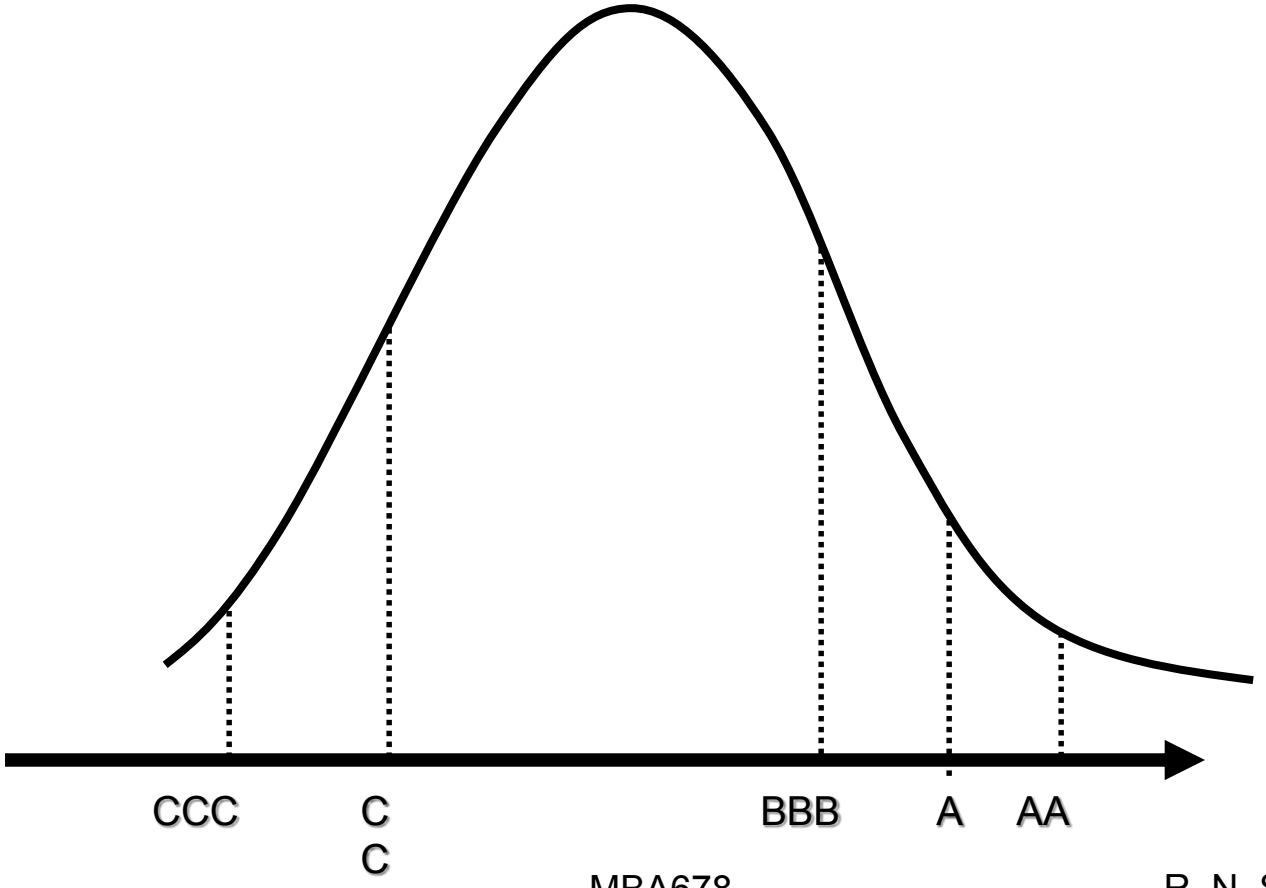
- It is not a symmetric matrix.
- The addition of rows should add up to 1.
- If we are given A, the transition matrix for a year then we can find that the investment or company will move to a certain different rating at the end of year two by $A^2 = A*A$.
- If we are interested in finding the corresponding rating at the end of the n^{th} year, provided we are given with the first year transition matrix only, then we have the transition matrix as $A^n = A*A*….$n times.
- It is always better to have the transition matrix for a short duration of time, e.g., weeks, months etc.
- Rather than extrapolating the information from the 1^{st} year to the n^{th} year, we can proceed iteratively such that we find the 2^{nd} year transition matrix and then after calculating the actual 2^{nd} year transition matrix we can find that for the 3^{rd} year and so on.

Asset Liability Management

One of the main questions is how to incorporate the effect of the correlation between the credit quality of the assets in the portfolio. Assuming that the distribution of borrower's risk is normal, we can characterize the relationship between the risks of two different borrowers using correlation coefficient.

Asset Liability Management

Depiction of the credit rating, considering normal distribution



Asset Liability Management

By quantifying these risks we are interested in capturing the main effects of these risks and the magnitude of the same on the profitability which is the main focus of banks. Following the regulators focus on valuing risk as a capital charge, model designers developed risk models aimed at the quantification of potential losses arising from each source of risk. The central concept of such models is the well known VaR, i.e, **Value at Risk**. Briefly stated VaR is a potential loss due to a defined risk. In other words VaR measures the worst expected loss under normal market conditions over a specific time interval at a given confidence level. Suppose that a portfolio manager has a daily VaR equal to 5 crores at 2%. This means that there is only two chance in 100 that a daily loss bigger than 5 crores occurs under normal market conditions.

Value at Risk (VaR)

The Value at Risk (VaR) is a potential loss based on present value. Potential losses can theoretically extend to the value of the entire portfolio. In other words VaR is the maximum loss at a present confidence level. The confidence level is the probability that the loss extends this upper bound. We must remember that VaR applies to all risks. For determining VaR we require to model the distributions of the values at some future time point, in order to define various loss percentiles, each corresponding to a confidence level. The VaR methodology serves to define risk based capital, or economic capital. Economic capital or risk capital is the capital required to absorb potential unexpected losses at the present confidence level. The confidence level reflects the risk appetite of the bank. In other words the confidence level is equivalent to the default probability of the bank.

Value at Risk (VaR)

The advantages of VaR are

- It provides a complete view of the portfolio.
- Captures the downside risk, which is more important, than the upside risk.
- Assigns monetary value to the risk, hence it can be quantified and easily understood.

Earning at Risk (EaR)

It is a simple and practical version of VaR and is called EaR. It measures, at present confidence levels, the potential adverse deviations of earnings. EaR shares the same underlying concept and has the benefit of being relatively easy to measure. Although similar to VaR, EaR does not relate the adverse deviations of earnings to the underlying risks because EaR aggregates the effect of all risks. By contrast, VaR requires linking the losses for each risk. Relating risks measures to the sources of risks is a prerequisite for risk management. Remember VaR provides the measure of economic capital, defined as an upper bound of future potential looses.

Value at Risk (VaR)

Basically VaR encompasses all of the following risks, which are Market risks, Credit risks, Interest rate risks, Other risks, etc.

Now there are several types of potential losses, which are:

- Expected losses (EL)
- Unexpected Losses (UL) = (VaR)
- Exceptional Losses

Expected Loss (EL)

It is the upper bound of loss not exceeded in more than a limited given fraction of all outcomes. This is a loss that we will face it sooner or later. Therefore, it makes sense to deduct EL from revenues, since it represents an overall average charge. If there were no random deviations around this average, there would be no need to add capital to economic provisions. This rationale implies that capital should be in excess of expected loss under economic provisioning.

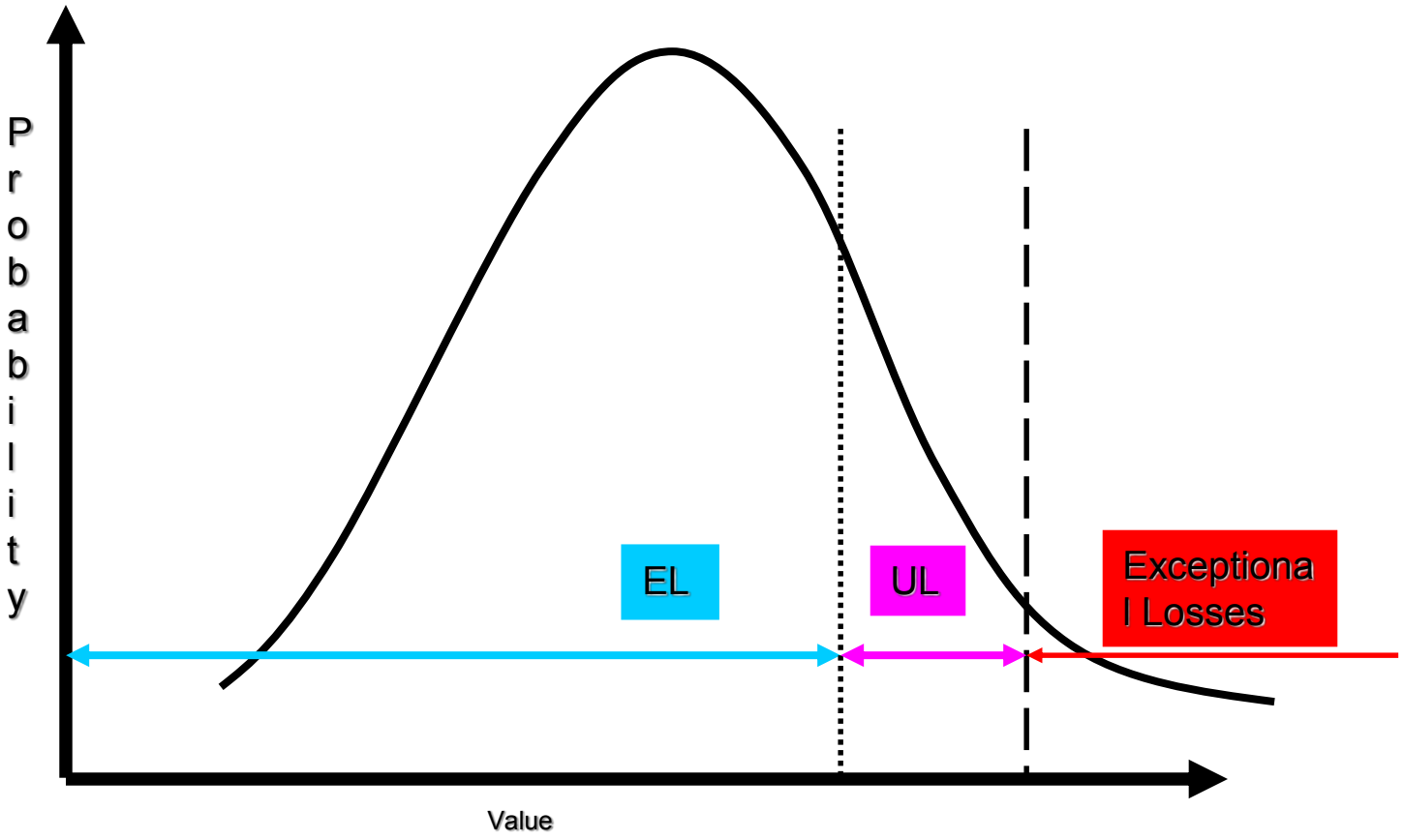
Unexpected Loss (UL) = VaR

These are potential losses in excess of the expected value. The loss percentile is the upper bound of loss not exceeded in more than a given fraction of all possible cases, this fraction being the confidence interval. It is L(α), where α is the one tailed probability of exceeding L(α). For example L(1%) = 100 means that the loss exceeds the value of 100 in no more than 1% of cases (one out of 100 possible scenarios, or two to three days within a year). The purpose of VaR model is to provide the loss distribution, or the probability of each loss value, to derive all loss percentiles for various confidence levels. The unexpected loss is the excess of the loss percentiles over the expected loss, L(α) – EL. Economic capital is equal to unexpected loss measured as a loss percentile in excess of expected loss (under economic provisioning).

Exceptional Loss

These are losses in excess of the sum of the EL plus the UL. Only under stress condition can we ever think of calculating this value..

How are the losses defined



Value at Risk (VaR)

VaR is used across the board and definition of VaR can also be stated as follows

We are X percent certain that we will not lose more than V amount of money in the next N days. This value V is the VaR.

Before we embark on finding the VaR we must remember that the volatility of any financial assets is calculated daily, so we need to convert this volatility to an yearly basis for VaR calculations.

Thus $\sigma_{yearly} = \sigma_{daily} * 365^{1/2}$ or if the volatility is given on an weekly basis then $\sigma_{yearly} = \sigma_{weekly} * 52^{1/2}$

Value at Risk (VaR)

Assume the volatility of Infosys is 5% per day and also consider a portfolio with Rs. 10,00,000 in shares of Infosys. We are interested in finding the VaR for 10 days at 99% confidence level.

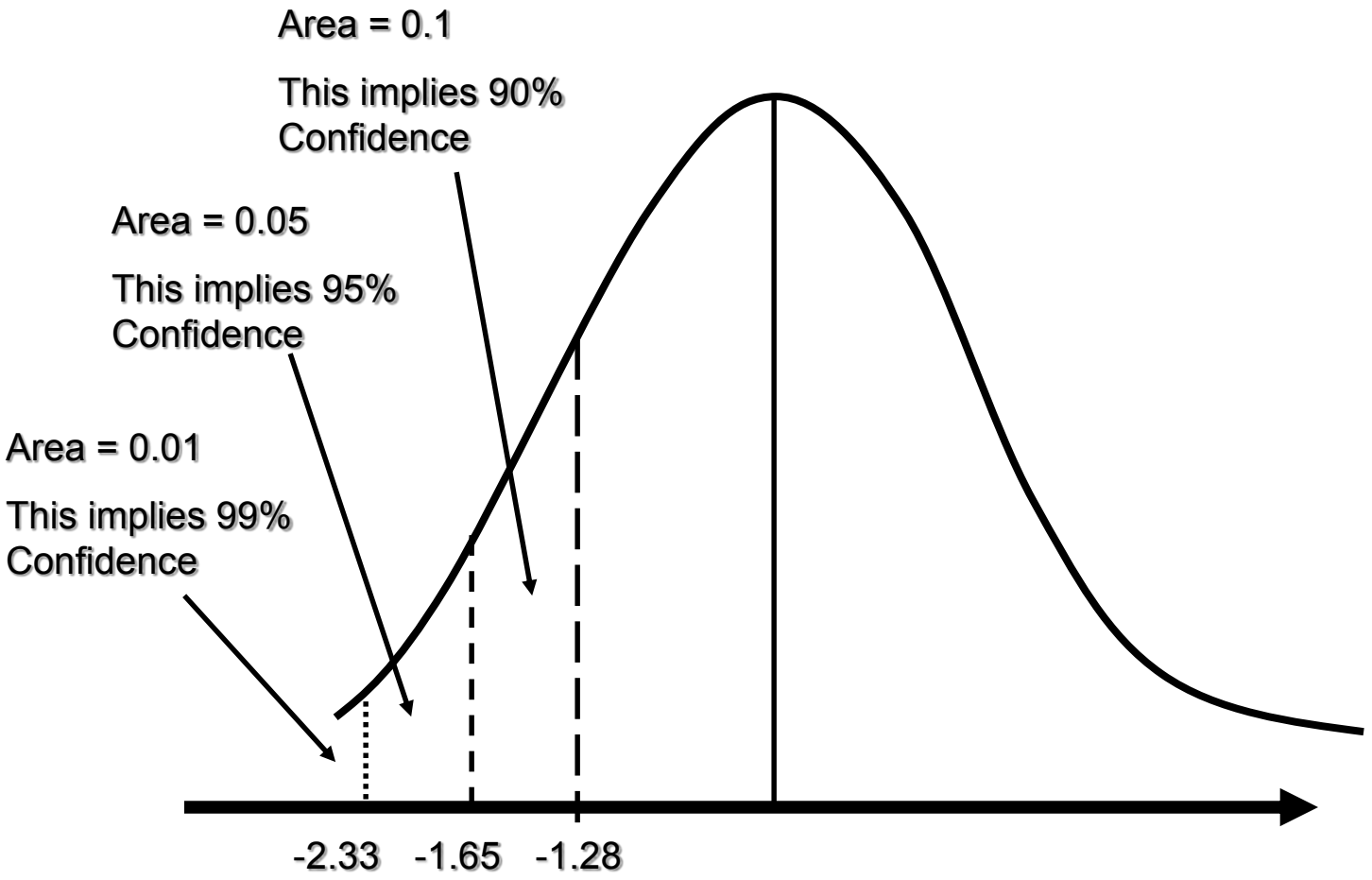
Now the standard deviation for the daily changes of price of Infosys changes the value of the position by Rs. 50,000 daily. Assume that changes between days are independent (this is also a very naïve assumption which is not true practically), we see that the change in the position for these 10 days is Rs. 1,58,114.

Value at Risk (VaR)

This basically means that the average change in the value of the portfolio is zero (an assumption under VaR) with a volatility of Rs. 1,58,114. Now considering 99% confidence level we say that VaR=2.33*1,58,114=3,68,405.

- What is the VaR for 95% confidence level?
- What is the VaR for 90% confidence level?

Value at Risk (VaR)



Value at Risk (VaR)

Note:

- 2.33 means that there is a 1% probability that a normally distributed variable will decrease by more than 2.33 standard deviation.
- 1.65 means that there is a 5% probability that a normally distributed variable will decrease by more than 1.65 standard deviation.
- 1.28 means that there is a 10% probability that a normally distributed variable will decrease by more than 1.28 standard deviation.

Value at Risk (VaR)

Assume the volatility of TISCO is 3% per day and also consider a portfolio with Rs. 5,00,000 in shares of TISCO. We are interested in finding the VaR for 10 days at different confidence levels.

- Find the VaR at 99% confidence level. The answer is Rs. 1,10,527.
- Find the VaR at 95% confidence level.
- Find the VaR at 90% confidence level.

Value at Risk (VaR)

We will now consider two assets which comprise the portfolio. Then the natural question is how do we calculate the corresponding VaR. Do we find them individually and add then up or find the cumulative VaR.

Now we know that:

$$\sigma^2_{X+Y} = \sigma^2_X + 2*\text{cov}(X,Y) + \sigma^2_Y$$

$$\sigma^2_{X+Y} = \sigma^2_X + 2*\rho_{X,Y}*\sigma_X*\sigma_Y + \sigma^2_Y$$

Value at Risk (VaR)

Let us continue with the example we were considering before, assuming a correlation coefficient of 0.75 above.

Hence to find the VaR for the portfolio we first find the 10 day change of the portfolio due to the individual volatilities, which his given below:
$\{(0.05*10^{1/2}*10*10^{5})^{2}+2*0.75*(0.05*10^{1/2}*10*10^{5})*(0.03*10^{1/2}*5*10^{5})+(0.03*10^{1/2}*5*10^{5})^{2}\}^{1/2}$.

Thus VaR at 99% confidence level is found out by multiplying this value with 2.33.

General properties of any risk measures

1) Positive homogeneity: $\rho(\lambda x) = \lambda\, \rho(x)$, $\forall$ r.v. x and $\lambda \in R^+$
2) **Sub-additivity: $\rho(x + y) \leq \rho(x) + \rho(y)$, $\forall$ r.v. x and y**
3) **Monotonicity: $x \leq y \Rightarrow \rho(x) \leq \rho(y)$, $\forall$ r.v. x and y**
4) Transitional invariance: $\rho(x + r_f) = \rho(x) - \alpha$, $\forall$ r.v. x, $\lambda \in R^+$, r_f is the risk less interest rate

General properties of any risk measures

1) The first property means that the risk for a portfolio, consisting of two or more assets, should be less than the addition of the risks of the individual assets. If this is not true then it makes no sense for any investor to go for **diversification**.
2) Simply stated the second property means that more the return, more would be the risk for any asset or portfolio.

Value at Risk (VaR)

VaR generally does not follow the above two important properties we had just discussed. The main problem is that VaR cannot be considered as a linear addition. It is non-linear in nature and hence we have to use complicated mathematics to find the exact values of VaR for a portfolio, if it consists of more than one asset where each of the assets have some correlation coefficient existing between them. The main assumption for the problem we have just considered, assuming two assets, viz, Infosys and TISCO, was, that the joint distribution for the portfolio distribution is also normal, which is definitely not true and is a very simplistic assumption .